basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

UMALUSI (APPROVED)
15-11-2023

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

APPROVED
C. BROWNELL (DBE)-(UM)
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Approved
2023-11-15

DEPARTMENT OF BASIC EDUCATION
PRIVATE BAG X895, PRETORIA 0001
2023 -11- 15
APPROVED MARKING GUIDELINE
PUBLIC EXAMINATION

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG 1*

Accept
A = -23,8 = -23,85 = -24 or -23,9
B = 0,2 = 0,22 or = 0,23

1.1	$a = -23,846...$ – Incorrect A+B Max 1 mark $b = 0,227...$ – Correct A and B but swapped $\hat{y} = -23,85 + 0,23x$ Max 1 mark. – No penalty for rounding off – Answer only full marks	✓ $a = -23,846...$ ✓ $b = 0,227...$ ✓ equation (3)
1.2	$\hat{y} = -23,85 + 0,23(550)$ CA $y = 102,65$ $y = 103$ Accept. **OR** $y = 101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101,02$ (calculator) (2)
1.3	$r = 0,98$ No penalty for rounding off	✓ $r = 0,98$ (1)
1.4	Very strong positive correlation / Strong positive / almost perfect (CA from 1.3)	✓ strong positive (1)

50	100	130	150	180	190	200	200

1.5.1 CA	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ CA **OR** $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2 CA	$\sigma = 50,50$ No penalty for rounding off	✓ $\sigma = 50,50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ $\therefore$ 1 stop Swapping σ and $\bar{x}$ = -ve answer. Final answer as No stop, 1 mark. Answer only = 2 marks if both $\bar{x}$ and σ are correct.	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG 2*

2.1

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \le x < 2$	5	5
$2 \le x < 4$	15	20
$4 \le x < 6$	13	33
$6 \le x < 8$	5	38
$8 \le x < 10$	2	40

✓ 5; 20
✓ 40 (2)

2.2 40 staff members CA (last value in cum freq column) ✓ answer (1)

2.3 33 staff members CA from 2.1. ✓ answer (1)

2.4

$$\bar{x} = \frac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times 15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times 5)+(9\times 2)}{40+k} = 4$$

✓ answer from Q2.2 + k
✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$
✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$

$$5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18 = 160+4k$$
$$3k+168 = 160+4k$$
$$k = 8$$ ✓

Answer only = 1 mark
CA from 2.2.

✓ answer (4)

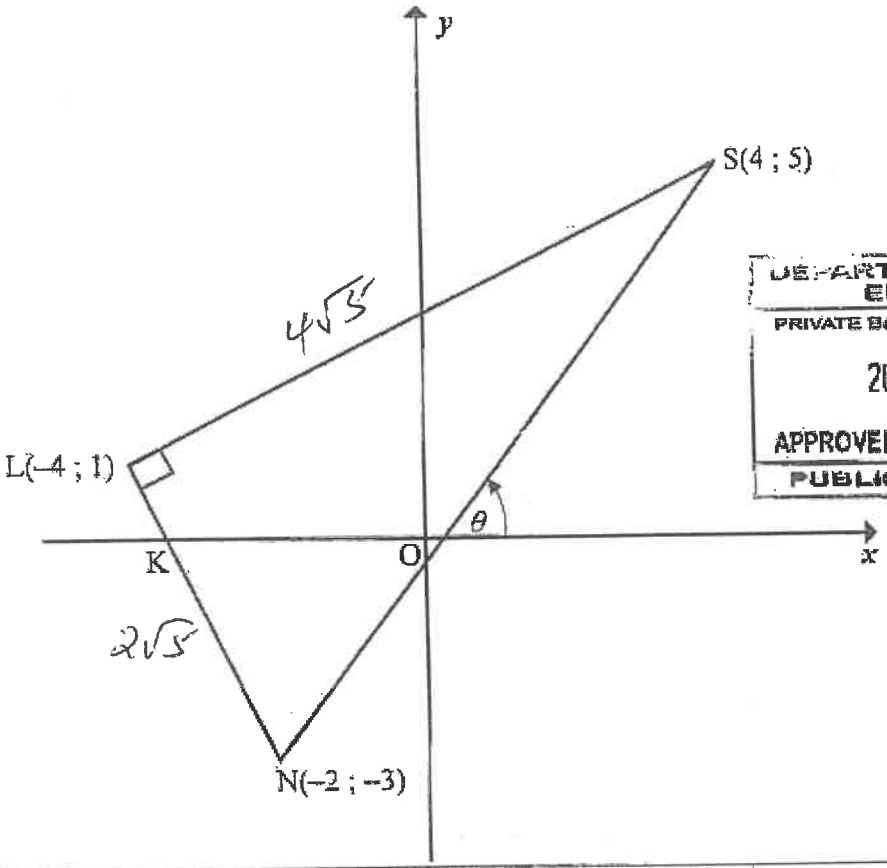
OR

$$\bar{x} = \frac{(1\times 5)+(15\times 3)+(13\times 5)+(5\times 7)+(2\times 9)}{40}$$
$$= 4{,}2$$
$$\bar{x}_{old} - \bar{x}_{current} = 4{,}2 - 4$$
$$= 0{,}2$$
$$\therefore 0{,}2 \times 40$$
$$= 8 \text{ teachers}$$

✓ 4,2
✓ $\bar{x}_{old} - 4$
✓ difference
✓ answer (4)

[8]

QUESTION/*VRAAG 3*



3.1	$SL=\sqrt{(x_S-x_L)^2+(y_S-y_L)^2}$ $SL=\sqrt{(4-(-4))^2+(5-1)^2}$ Answer only full Marks $SL=\sqrt{80}=4\sqrt{5}=8,94$ units Accept the decimal	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN}=\frac{5-(-3)}{4-(-2)}$ for interchanging the denominator, penalise and give 1 mark. $m_{SN}=\frac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m=\tan\theta=\frac{4}{3}$ CA. $\theta=53,13°$	✓ $\tan\theta=m_{SN}$ ✓ answer (2)
3.4	$m_{LN}=\frac{1-(-3)}{-4-(-2)}$ The gradient = 0 Mark. Answer only = 1 Mark $m_{LN}=-2$ $L\hat{K}O=116,565...°$ $L\hat{N}S=116,565...°-53,13°$ CA. $L\hat{N}S=63,44°$	✓ $m_{LN}=-2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR $L\hat{N}S = \tan^{-1}(-2) - \tan^{-1}(\frac{4}{3})$ $= 116,565^\circ - 53,13^\circ$ $= 63,44^\circ$	
	$SN = 10$ units $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63,44^\circ$	✓ $SN = 10$ units ✓ correct trig ratio ✓ answer (3)
	OR	
	$LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \frac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63,44^\circ$	✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
	OR	
	$SN = 10$ units ✓ (1) $LN = 2\sqrt{5}$ units ✓ (1) $\cos L\hat{N}S = \frac{2\sqrt{5}}{10}$ ✓ $L\hat{N}S = 63,44^\circ$ ✓	✓ $SN = 10$ units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ CA from 3.2. $1 = \frac{4}{3}(-4) + c$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $c = \frac{19}{3}$ Stopping at c 2 marks $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ CA from 3.3 Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ CA $= 20$ units^2	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
	OR	

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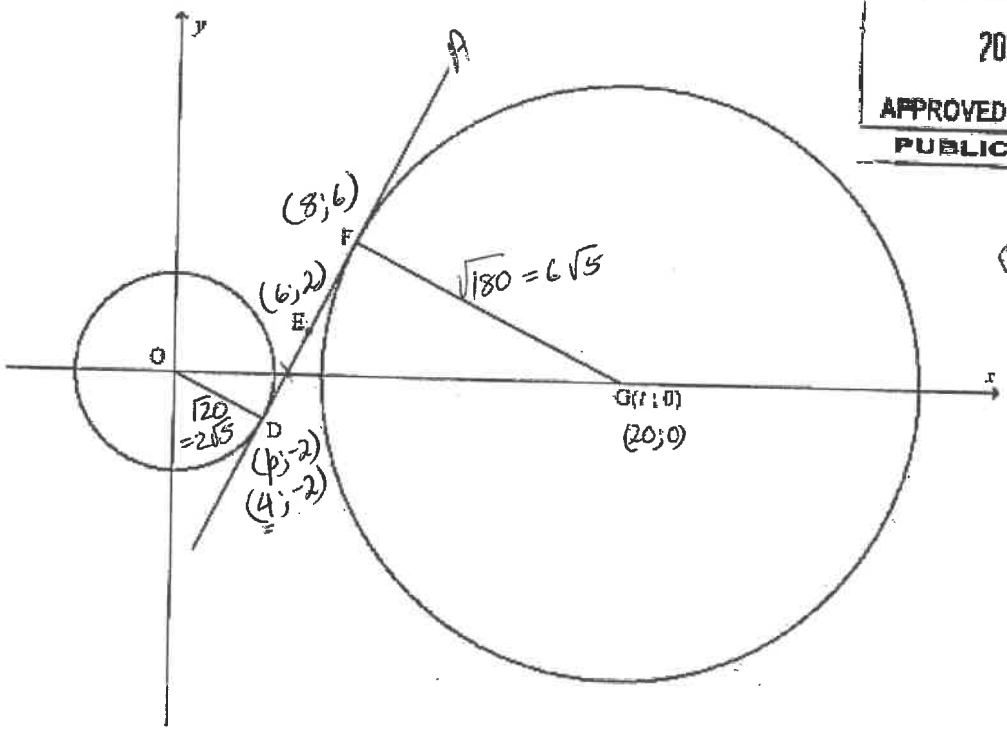
Also cosine rule.

	$SN = 10$ units $LN = \sqrt{(-4-(-2))^2+(1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ $= 20$ units^2	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle $= P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $= P(1;1)$ **OR** Let the coordinates of P be $(a;b)$. Then, PL = PN: $(-4-a)^2+(1-b)^2 = (-2-a)^2+(-3-b)^2$ $a-2b=-1$ .....equation 1 If PS = PN, then: $4a+2b=6$ ..... equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$ **OR** If PL = PN, then: $a-2b=-1$ .....equation 1 If PS = PL, then: $2a+b=3$ .....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$	✓ SN is a diameter of circle S, L, N ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
3.8	$L\hat{P}N = \theta = 53,13°$ [alt ∠s; LP || x–axis] $\therefore L\hat{P}S = 126,87°$ **OR** $L\hat{N}S = 63,44°$ $\therefore L\hat{P}S = 126,88°$ [∠ at centre = 2 × ∠ at circumference] **OR** $L\hat{S}N = 26,56°$ [sum of ∠s in Δ] $S\hat{L}P = 26,56°$ [∠s opp equal radii] $\therefore L\hat{P}S = 126,88°$ [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ $\therefore L\hat{P}S = 126,87°$	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

D4.1/5489 $\frac{6-(-2)}{8-4} = \frac{6+2}{4} = 2$

QUESTION/*VRAAG 4*



NB F (5;0) ; (4;-2)
$m_{FD} = \frac{-2-0}{4-5} = 2$

4.1	D(p ; −2) $x^2 + y^2 = 20$ ✓ $p^2 + (-2)^2 = 20$ Can also write x^2. etc. $p^2 = 16$ ✓ $p = \pm 4$ $p = 4$	✓ substitution of point D(p ;−2) ✓ $p^2 = 16$ Full mark (2)
4.2	$\frac{4+x_F}{2} = 6$ $\quad \frac{-2+y_F}{2} = 2$ ✓ $x_F = 8$ ✓ $\quad y_F = 6$ ✓ F(8;6) Answer only full Marks **OR** $x_E - x_D = 6-4$ $\quad y_E - y_D = 2-(-2)$ ✓ $= 2$ $\quad = 4$ $x_F = 6+2 = 8$ ✓ $\quad y_F = 2+4 = 6$ ✓ F(8;6)	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

D4|5489

4.3	$m_{DE} = \frac{-2-2}{4-6}$ CA from 4.2. $\frac{\Delta x}{\Delta y}$ inverse $m_{DE} = 2 \rightarrow$ m calculated easide = 2 Marks $-2 = 2(4) + c$ ✓ **OR** $y-(-2) = 2(x-4)$ ✓ $c = -10$ full mark if left at $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$ **OR** Wrong: one m but right application of rad ⊥ tangent rule 2 Marks. $m_{DD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius] $-2 = 2(4) + c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ CA from 4.3. $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ **OR** $y = 2x-10$ $0 = 2x-10$ $x = 5$ ✓ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

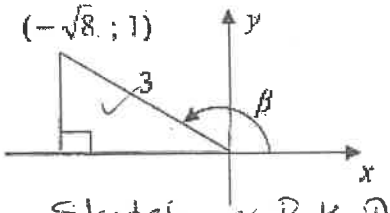
D41/5489

4.5	F(8;6) G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$ Let's not mind the arrangement but equated to 0 must be	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units **OR** Smaller circle $r=2\sqrt{5}$ $k=2(2\sqrt{5})+20-8\sqrt{5}$ or $k=2(6\sqrt{5})+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ 6,58 33,42 $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ $=11{,}06$ units $=28{,}94$ units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4) **[20]**

QUESTION/VRAAG 5

5.1.1

$\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$



✓ $x^2 + y^2 = r^2$

Sketch or Pyth Theorem
- No sketch, No pyth Theorem 2 Marks.
- Diagram on quad 2 even if not labelled
- -ve x, 2 Marks

$x = -\sqrt{8} = -2\sqrt{2}$ ✓ $x = -2\sqrt{2}$

$\cos\beta$

$= \frac{-2\sqrt{2}}{3}$ ✓ answer (3)

OR

$\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$

$\cos^2\beta = 1 - \sin^2\beta$ ✓ square identity

$\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$

$\cos^2\beta = \frac{8}{9}$ ✓ $\cos^2\beta$

$\cos\beta = \frac{-\sqrt{8}}{3}$

$= \frac{-2\sqrt{2}}{3}$ ✓ answer (3)

5.1.2

$\sin 2\beta$ CA from 5.1.1.

$= 2\sin\beta\cos\beta$ ✓ double angle

$= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ ✓ substitution

$= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$

$= \frac{-4\sqrt{2}}{9}$ ✓ answer (3)

5.1.3

$\cos(450° - \beta)$ Answer only = 1 Mark.

$= \cos(90° - \beta)$ → If omitted, give 2 marks at sin β.

$= \sin\beta$ ✓ $\cos(90° - \beta)$

$= \frac{1}{3}$ ✓ co-ratio

- Not changing to co-ratio 1 Mark.

✓ answer (3)

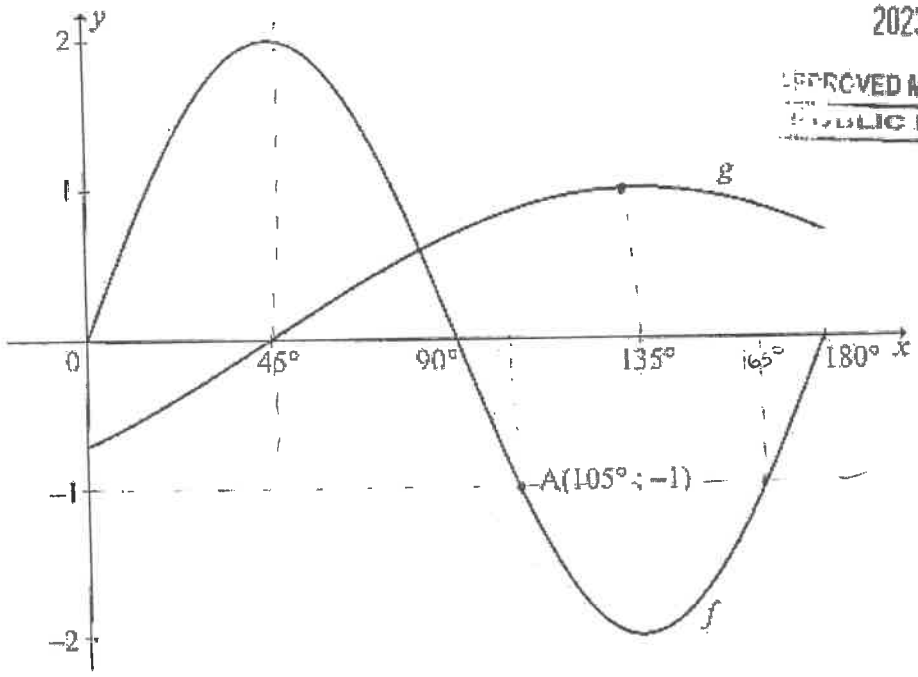
OR

	$\cos(450°-\beta)$ $=\cos 450°\cos\beta+\sin 450°\sin\beta$ $=\cos 90°\cos\beta+\sin 90°\sin\beta$ $=\sin\beta$ $=\frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	$\text{LHS}=\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $=\frac{\cos^2 x\left(\cos^2 x+\sin^2 x\right)}{1+\sin x}$ $=\frac{1-\sin^2 x}{1+\sin x}$ $=\frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $=1-\sin x.$ $=\text{RHS}$	✓ factors ✓ $\sin^2 x+\cos^2 x=1$ ✓ $\cos^2 x=1-\sin^2 x$ ✓ factors
	OR	(4)
	$\text{LHS}=\frac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $=\frac{\cos^4 x+\left(1-\cos^2 x\right)\cos^2 x}{1+\sin x}$ $=\frac{\cos^4 x+\cos^2 x-\cos^4 x}{1+\sin x}$ $=\frac{1-\sin^2 x}{1+\sin x}$ $=\frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $=1-\sin x$ $=\text{RHS}$	✓ $\sin^2 x=1-\cos^2 x$ ✓ expansion ✓ $\cos^2 x=1-\sin^2 x$ ✓ factors
	OR	(4)
	$\text{RHS}=1-\sin x$ $=(1-\sin x)\times\frac{1+\sin x}{1+\sin x}$ $=\frac{1-\sin^2 x}{1+\sin x}$ $=\frac{\cos^2 x}{1+\sin x}$ $=\frac{\cos^2 x\left(\sin^2 x+\cos^2 x\right)}{1+\sin x}$ $=\frac{\cos^4 x+\cos^2 x.\sin^2 x}{1+\sin x}$ $=\text{LHS}$	✓ $\times\frac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1-\sin^2 x=\cos^2 x$ ✓ $1=\cos^2 x+\sin^2 x$ (4)

5.2.2	$\sin x+1=0$ $\sin x=-1$ ref. $\angle=90°$ $x=270°$	for both 90° & 270°; 1 mark Answer only = 2 marks. Ending at the Gen Sol: 1 Mark $x\neq 270$: 1 Mark.	✓ $\sin x+1=0$ ✓ $x=270°$ (2)
5.2.3	$y=\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $y=1-\sin x$ $\therefore$ Minimum $=0$ $\quad y=0$	0. 2Marks	✓✓ Minimum $=0$ (2)
5.3.1	$\sin(A-B)$ $=\cos[90°-(A-B)]$ $=\cos[(90°-A)-(-B)]$ $=\cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $=\sin A\cos B+\cos A(-\sin B)$ $=\sin A\cos B-\cos A\sin B$	With (+B) = 1mark $\cos[(90°-A)+B]$ 1 Mark Interchanging A and B correctly 3Marks	✓ co-ratio ✓ compound angle ✓ reduction (3)
	OR $\sin(A-B)$ $=\cos[90°-(A-B)]$ $=\cos[(90°+B)-A]$ $=\cos(90°+B)\cos A+\sin(90°+B)\sin A$ $=-\sin B\cos A+\cos B\sin A$ $=\sin A\cos B-\cos A\sin B$	DEPARTMENT OF BASIC EDUCATION PRIVATE BAG X895, PRETORIA 0001 2023 -11- 15 APPROVED MARKING GUIDELINE PUBLIC EXAMINATION	✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\sin(48°-x)=\sin(90°-2x)$ (I) $48°-x=90°-2x+k.360°$ or (II) $48°-x=180°-(90°-2x)+k.360°$ $x=42°+k.360°$ $\quad -3x=42°+k.360°$ $x=-14°-k.120°$; $k\in Z$	No KEZ: -1mark from the total Marks	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5)
	OR $\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin(48°-x)=\cos 2x$ $\cos(90°-48°+x)=\cos 2x$ $\cos(42°+x)=\cos 2x$ $42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$ $-x=-42°+k.360°$ $\quad 3x=318°+k.360°$ $x=42°-k.360°$ $\quad x=106°+k.120°$; $k\in Z$		✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5)

5.4	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\dfrac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$	✓ $3x=(2x+x)$
	$=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$	✓ expansion ✓ double angle of cos2x
	$=\dfrac{2\sin 2x\cos x}{2\cos^2 x}$	✓ simplification
	$=\dfrac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\dfrac{4\sin x\cos^2 x}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
	OR	
	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\dfrac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$	✓ $3x=(2x+x)$ ✓ double angle of cos2x
	$=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ expansion
	$=\dfrac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\dfrac{\sin x(2\cos^2 x+\cos 2x+1)}{2\cos^2 x}$	✓ common factor
	$=\dfrac{\sin x(2\cos^2 x+2\cos^2 x-1+1)}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
		[31]

QUESTION/*VRAAG 6*

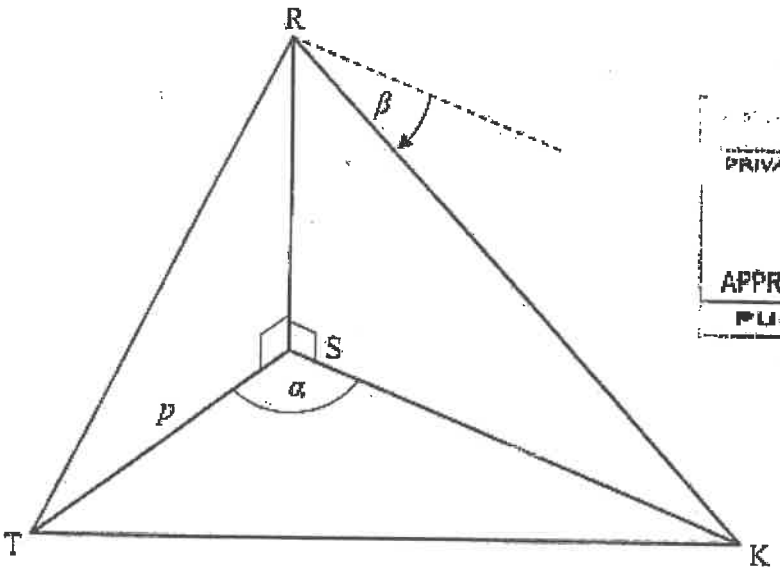


6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ **OR** $y \in [-0,71; 1]$ **OR** $-\frac{\sqrt{2}}{2} \leq y \leq 1$ ✓ CA if the y-int is between 0 and -1. End points correct, notation incorrect 1 Mark	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ (2)
6.3.1	$x \in (45°; 90°)$ **OR** $45° < x < 90°$ Including end points : 1 Mark From 45 to 90 = 1 Between 45 & 90 = 2	✓✓ $x \in (45°; 90°)$ (2)
6.3.2	$f(x)+1 \leq 0$ $f(x) \leq -1$ $x \in [105°; 165°]$ **OR** $105° \leq x \leq 165°$ Answer only = 2 Marks Wrong notation = 1 Mark " endpoints = 0; irrespective of correct notation	✓✓ $x \in [105°; 165°]$ (2)
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ ✓ $k = 15°$ ✓ or $k = 75°$ ✓ Answer only = 3 Marks Only [15; 75] = 2 Marks $k = \{15°; 75°\}$ Interval Notation: $15° < x < 75°$ 2 Marks	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$ Answer only: 2 Marks.	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

9 Mark: $x > 5$ or $x < 90$

D41|5489

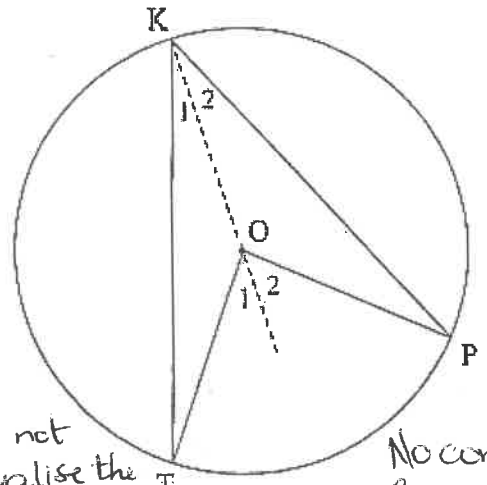
QUESTION/*VRAAG 7*



7.1	Area $\Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ any of the two answers $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ ÷ Cosβ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $R\hat{K}S = \beta$ ✓ correct trig ratio (2) ✓ $R\hat{K}S = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ * 2 Steps = 3 Marks $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,803932184$ $\alpha = 53,51°$ *	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

QUESTION/*VRAAG 8*

8.1



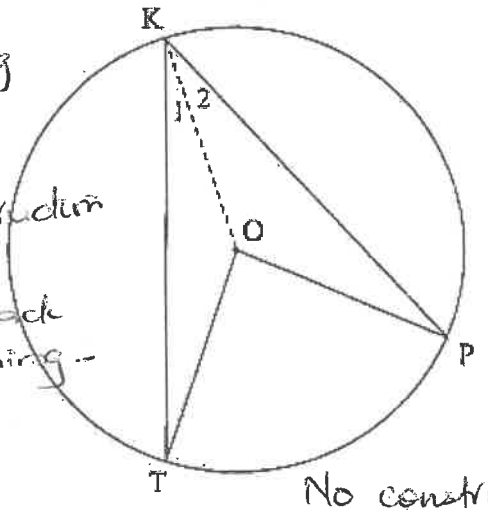
If ~~not~~ given in words but not in the diagram, visualise the construction.

No construction, no statement = 0 Mark. Base angles not accepted.

8.1	Construction: Draw KO produced	✓ construction
	$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ]	
	But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides]	✓ S / R
	$\therefore \hat{O}_1 = 2\hat{K}_1$	✓ S
	$\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ]	
	But $\hat{K}_2 = P$ [∠s opp equal sides]	
	$\therefore \hat{O}_2 = 2\hat{K}_2$	✓ S
	$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2(\hat{K}_1 + \hat{K}_2)$	✓ S
	$\therefore T\hat{O}P = 2T\hat{K}P$	(5)
	OR	

8.1

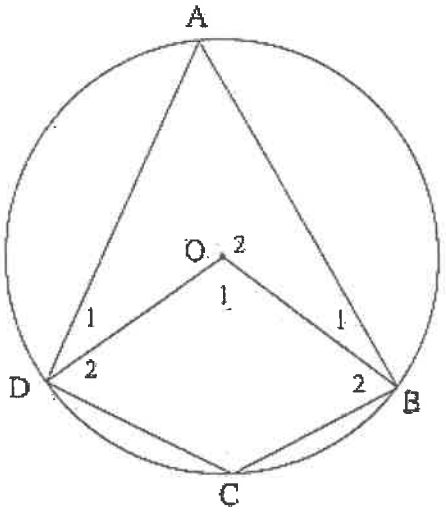
- Changing naming in the diagram 4 Marks.
- If there's no construction breakdown.
- But if he goes back to given naming - 5 Marks



No construction – Breakdown

8.1	Construction: Draw KO		✓ construction
	$\hat{T} = \hat{K}_1$	[∠s opp. equal sides]	✓ S / R
	$\therefore K\hat{O}T = 180° - 2\hat{K}_1$	[sum of ∠s of Δ KOT]	✓ S
	$\hat{P} = \hat{K}_2$	[∠s opp. equal sides]	
	$\therefore K\hat{O}P = 180° - 2\hat{K}_2$	[sum of ∠s of Δ KOP]	✓ S
	$T\hat{O}P = 360° - (K\hat{O}T + K\hat{O}P)$	[∠s around a point]	
	$= 360° - (180° - 2\hat{K}_1 + 180° - 2\hat{K}_2)$		✓ S
	$= 2\hat{K}_1 + 2\hat{K}_2$		
	$= 2(\hat{K}_1 + \hat{K}_2)$		
	$\therefore T\hat{O}P = 2T\hat{K}P$		
			(5)

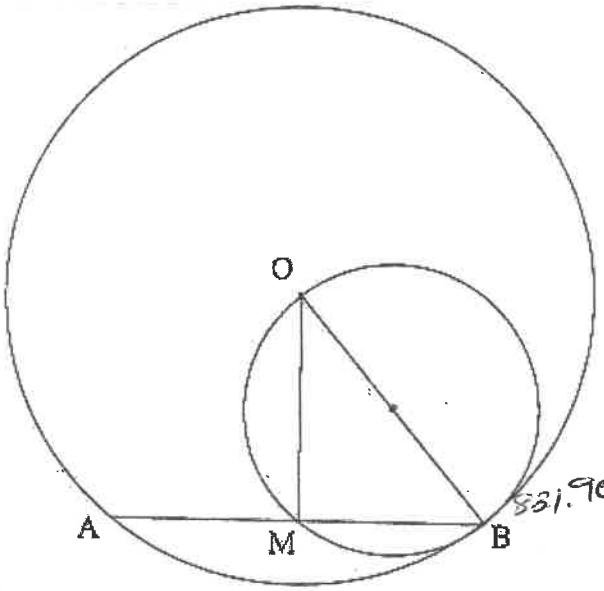
8.2



8.2	$\hat{O}_1 = 4x+100°$	[given]	
	$\therefore \hat{A} = 2x+50°$	[$\angle$ at centre = 2 $\times\angle$ at circumference]	✓ S ✓ R
	$x+34°+2x+50°=180°$	[opp $\angle$s of cyclic quad]	✓ S ✓ R
	$3x=96°$		
	$x=32°$		✓ answer
	OR		(5)
	$\hat{O}_2 = 2x+68°$	[$\angle$ at centre = 2 $\times\angle$ at circumference]	✓ S ✓ R
	$4x+100°+2x+68°=360°$	[$\angle$s round a pt]	✓ S ✓ R
	$6x=192°$		
	$x=32°$		✓ answer
	OR		(5)
	$\hat{O}_2 = -4x+260°$	[$\angle$s round a pt]	✓ S ✓ R
	$2\hat{C} = -4x+260°$	[$\angle$ at centre = 2 $\times\angle$ at circumference]	✓ S ✓ R
	$\hat{C} = -2x+130°$	If x is -ve, subst in $\hat{C}$ and $\hat{O}_2$ for a +ve answer = 3 Marks	
	$x+34° = -2x+130°$	for any +ve answer work out and CA.	
	$3x=96°$		
	$x=32°$		✓ answer
			(5)

If $\hat{A} = \hat{C}_1$ ⇝ Break down.
for $\hat{C}_1 = 2\hat{C}$ ⇝ Stop marking.

8.3

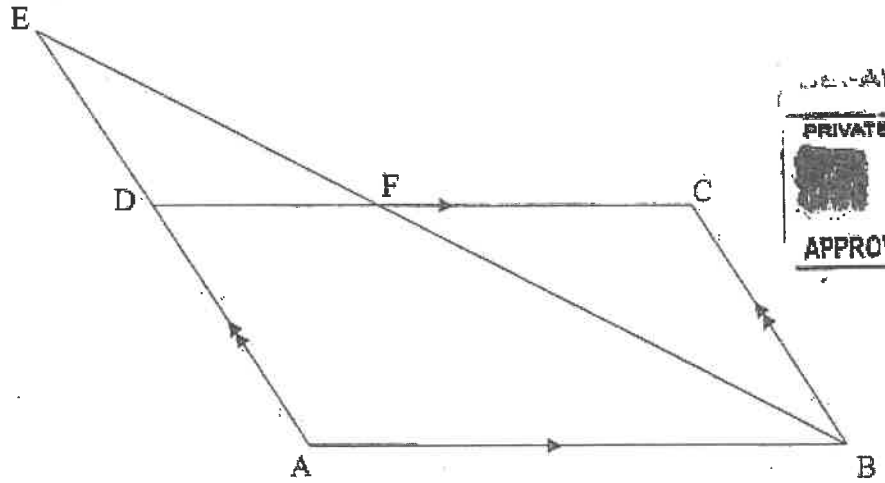


$\hat{O}MB = 90°$ on the diagram = 1 Mark.

Statement: 1 mark

8.3.1	$\hat{O}MB = 90°$ [$\angle$ in semi circle] ⊥ bisector = 0 Mark. OM ⊥ AB wrong reason.		✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$	[line from centre ⊥ to chord] (Rad ⊥ chord)	✓ S ✓ R
	$OB^2 = OM^2 + MB^2$	[Pythagoras]	
	$OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$		✓ S
	$OB = 10$ units		✓ answer (4)
			[16]

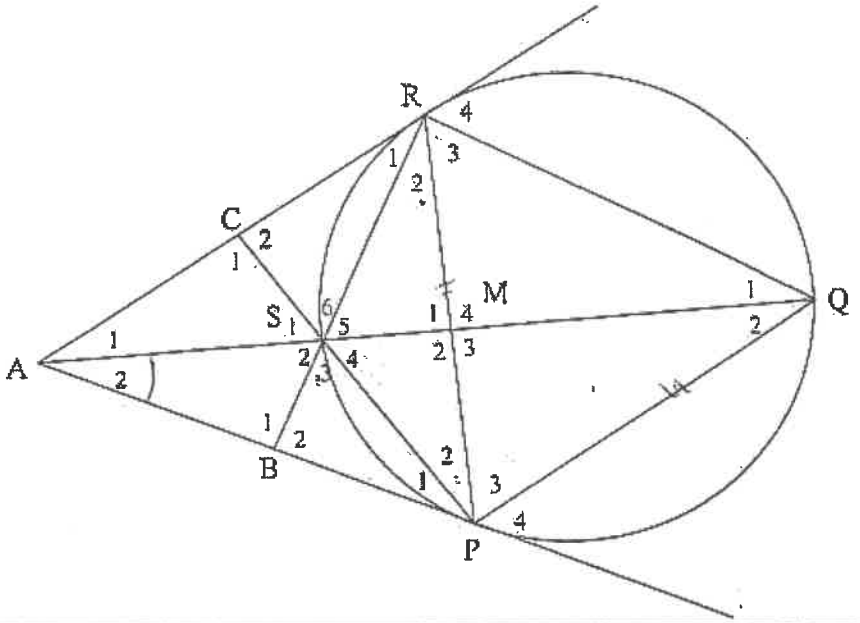
QUESTION/*VRAAG 9*



DE/14 = 3/7

9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ‖ AB] **OR** [line ‖ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units If no naming of // lines 1 mark for statement only.	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: Ê is common EDF = Â [corresp ∠s; EA ‖ CB] EFD = EBA [corresp ∠s; DC ‖ AB] ΔEDF ||| ΔEAB [∠;∠;∠] CA on // lines DC // AB	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [||| Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 14 – DF FC = 8 units [DC = AB = 14 units; opp sides of ||^{m}] CA for any wrong value of DF if subtracted from 14. **OR** ΔEDF ||| ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [||| Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ||^{m}] 3FC = 56 – 4FC FC = 8	✓ S ✓ DF = 6 ✓ FC = 14 – DF (3) ✓ ΔEDF ||| ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/*VRAAG 10*



10.1	$\hat{S}_3 = P\hat{Q}R$	[ext ∠ of cyclic quad]	✓ S ✓ R
	$\hat{R}_3 = P\hat{Q}R$	[∠s opp equal sides]	✓ S / R
	$\therefore \hat{S}_3 = \hat{R}_3$		
	But $\hat{S}_4 = \hat{R}_3$	[∠s in the same seg]	✓ S ✓ R
	$\therefore \hat{S}_3 = \hat{S}_4$		(5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$	[tan chord theorem]	✓ S ✓ R
	$\hat{S}_4 = P\hat{Q}R$	[proved in 10.1]	
	$\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$		✓ S
	SMRC is a cyclic quad	[converse ext ∠ of cyclic quad]	✓ R
		ext ∠ of a quad.	(4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$	[ext ∠ of Δ]	✓ S ✓ R
	$\hat{S}_4 = \hat{P}_1 + \hat{A}_2$	[ext ∠ of Δ]	✓ S
	$\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$		
	But $\hat{P}_1 = \hat{R}_2$	[tan chord theorem]	✓ S ✓ R
	$\therefore \hat{P}_2 = \hat{A}_2$		
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R
		OR	
		[∠ between line and chord]	
		OR	
		[converse alt seg theorem]	(6)
	OR		

	In ΔMSP and ΔMPA		
	$\hat{M}_2$ is common		✓ S
	AR = AP	[tans from same point]	✓ S / R
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$	[∠s opp equal sides] tangents	✓ S
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$	[proved in 10.2]	
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_3$		✓ S
	$\therefore \hat{P}_2 = \hat{A}_2$	[sum of ∠s in Δ]	✓ S
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R (6)
			[15]

TOTAL/TOTAAL: 150